Royal guards

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For easier implementation, let the castle be surrounded by walls. A *row segment* will be each continuous part of some row that doesn't contain a wall and cannot be extended in any direction (e.g. immediately to the left and to the right of it are walls). Similarily we define a *column segment*. Clearly each segment may contain at most one guard.

Let's build a bipartite graph where vertices of one partition correspond to the row segments, vertices of the other partition correspond to the column segments and two vertices are connected by an edge iff the corresponding row segment and column segment intersect and their intersection doesn't contain a pit. In other words the edges correspond to the squares where a guard may stand.

Now consider any valid placement of guards. Then the edges corresponding to places where guards stand form a matching in our graph – each row segment and each column segment contains at most one guard, and thus each vertex incides with at most one edge. On the other hand each matching in our graph corresponds to some valid placement of the guards.

In other words, to place the maximum number of guards into the castle, it suffices to find a maximum matching in the graph we constructed above. There is a well-known algorithm to do this in $O(|V|.(|V| + |E|))$ time, based on repeated construction of an augmenting path and exchanging matched vs. unmatched edges along it. As $|V|, |E| = O(MN)$, this algorithm has the time complexity $O(M^2N^2)$.